Lecture 2: Quantum Relays and Repeaters

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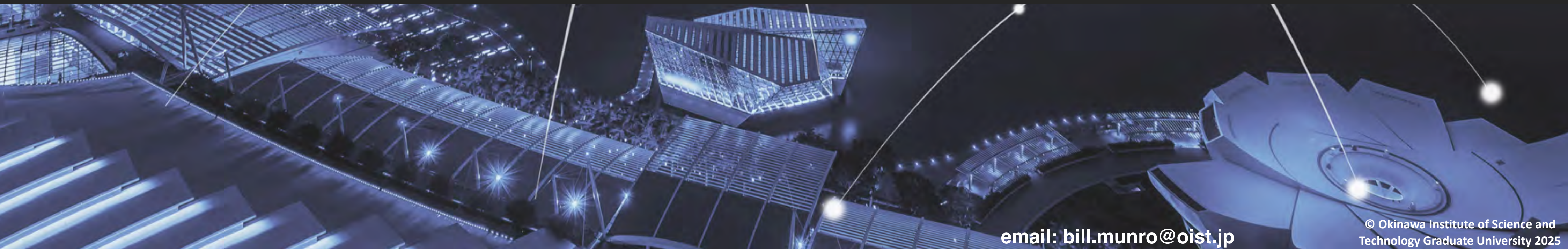

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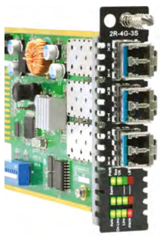
The classical world: losses in the channel

- The modern telecommunications industry solves this issue through **Optical amplifiers and optical repeaters**

- OA's are optoelectronic circuits that amplifies the distorted light signal (including noise) it receives back to its original amplitude before transmitted it further down the channel. Each time this amplification process is performed the signal to noise ratio gets worse until we reach a point at which the signal cannot be distinguished from the noise. This fundamentally limits the distance a signal can be transmitted over.



- Optical repeaters are optoelectronic circuits that regenerate the original signal through pulse reshaping, amplification and potentially resynchronisation leaving the signal being transmitted without the unwanted noise it has accumulated. Using optical repeaters allows communication over arbitrary distances.

The quantum world

Loss in the channel is an even bigger problem

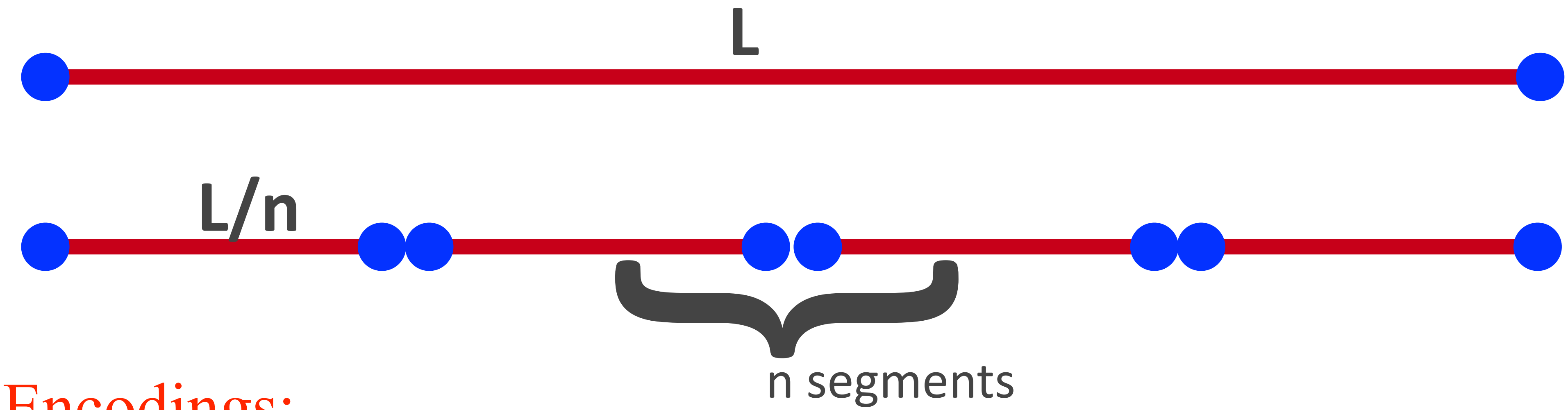
- The modern telecommunications industry solves this issue through **Quantum amplifiers and Quantum repeaters**
- Quantum amplifiers (QA) are well known and like their optical counterparts typically add excess noise to the signal, thus limiting the communication distance
- However noiseless linear amplifiers do exist (but unfortunately this are probabilistic (but heralded) in nature

- Quantum repeaters seem very different from their classical counterparts
- They tend to focus on the generation of entanglement between Alice and Bob
- However not only do they need to fix channel loss, they also need to be able to remove excess noise!!!!

- **Quantum repeaters require a classical network as well**

The Concept!!!

- We need to divide the channel in parts:
  - so take the long channel and …



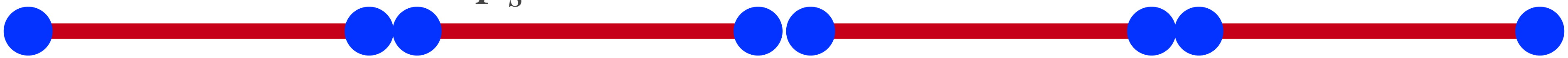
- Encodings:
  - Single-rail: using the presence or absence of a single photon in a single mode
  - Dual-rail: the presence of a photon in one or another mode as the qubit

The simplest case first

the entire channel $P_S = e^{-L/L_0}$

Number of attempts to succeed $N_S \sim e^{L/L_0}$

n segments $p_s = e^{-L/nL_0}$

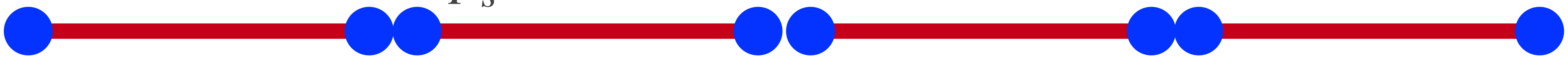


- How many attempts to succeed
  - without memories $n_S = (e^{L/nL_0})^n$
  - with memories $n_s \sim 100e^{L/nL_0}??$

L_0 is the attenuation length of the fiber

How???

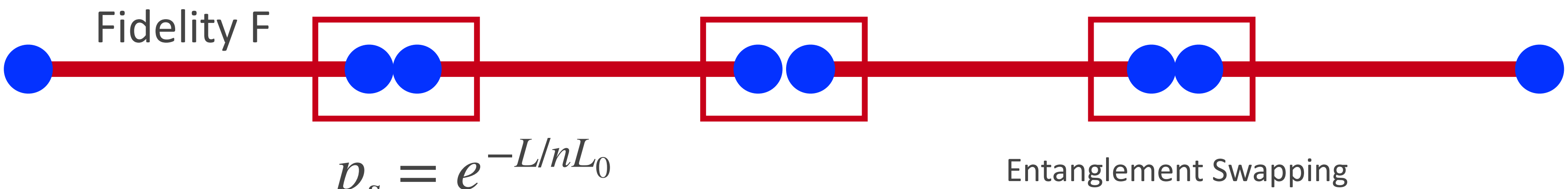
$$p_s = e^{-L/nL_0}$$



- Per segment we need on average $1/p_s$ attempts for it to be successful
- With say $100/p_s$ attempts exceptionally high probability of establishing the link.
- When successful we store in a quantum memory.

- **Perform attempts among all segments at the same time**
- likely all segments have a link within $100/p_s$ attempt
- join together to get the long link

Quantum Relay



$$p_s = e^{-L/nL_0}$$

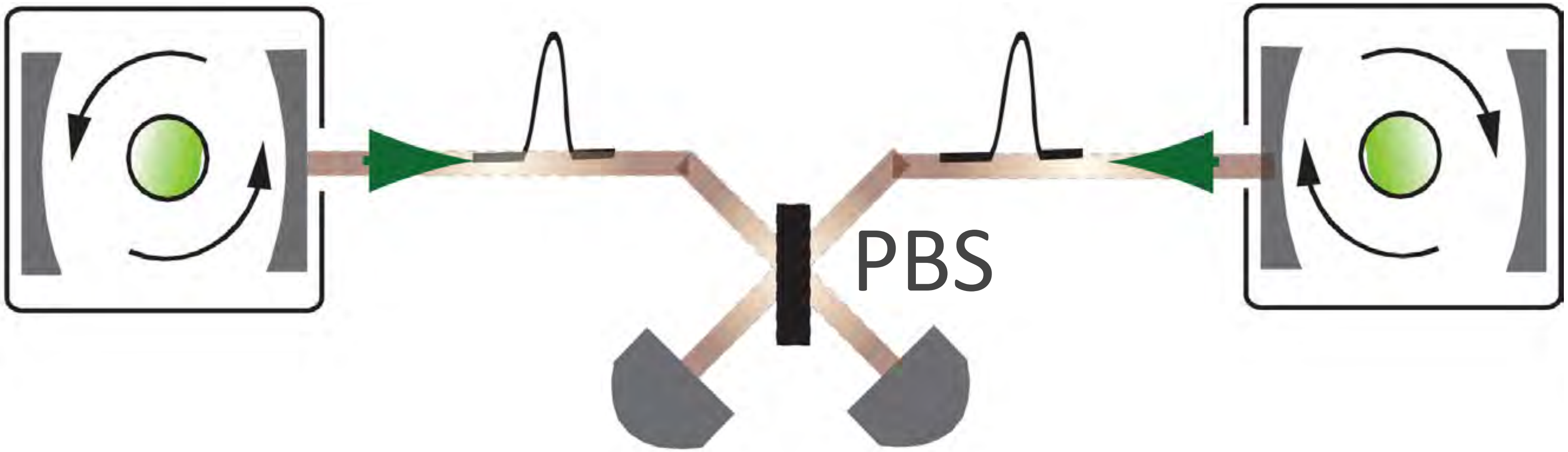
- **This allows us to establish long range entanglement efficiently**

Overview

- Introduce quantum entanglement distribution
- Introduce entanglement swapping
- Introduce entanglement purification
- Scaling of quantum relays and repeaters

Entanglement Distribution

- A mechanism to establish an entangled Bell state between adjacent quantum repeater nodes
- Let us look at this in a little more detail



- Consider we have a matter qubit memory that can emit a polarization photon of the

$$\frac{|g\rangle|H\rangle + |e\rangle|V\rangle}{\sqrt{2}}$$

Entanglement Distribution

$$\frac{|g\rangle|H\rangle + |e\rangle|V\rangle}{\sqrt{2}}$$

- Both matter qubit emit there polarization qubit
- So we have

$$\{|g\rangle|H\rangle + |e\rangle|V\rangle\} \otimes \{|g\rangle|H\rangle + |e\rangle|V\rangle\} =$$

$$|g\rangle|g\rangle|H\rangle|H\rangle + |g\rangle|e\rangle|H\rangle|V\rangle + |e\rangle|g\rangle|V\rangle|H\rangle + |e\rangle|e\rangle|V\rangle|V\rangle$$

PBS

$$|g\rangle|g\rangle|H\rangle|H\rangle + |g\rangle|e\rangle|HV\rangle|0\rangle + |e\rangle|g\rangle|0\rangle|HV\rangle + |e\rangle|e\rangle|V\rangle|V\rangle$$

$$|g\rangle|g\rangle|H\rangle|H\rangle + |e\rangle|e\rangle|V\rangle|V\rangle$$

Measure 1 photon at each detector in D or A basis

$$|g\rangle|g\rangle + |e\rangle|e\rangle$$ for DD, AA

What happens for DA, AD??

Entanglement Distribution: Success Probability

for generating

$$|g\rangle|g\rangle \pm |e\rangle|e\rangle$$

- This is effected by two factors:
  - Channel loss $\eta = e^{-L/L_0}$
  - Detection postselection

Overall success probability

$$P = \boxed{\sqrt{\eta}}\,\boxed{\sqrt{\eta}}\,\boxed{/2} = \eta/2$$

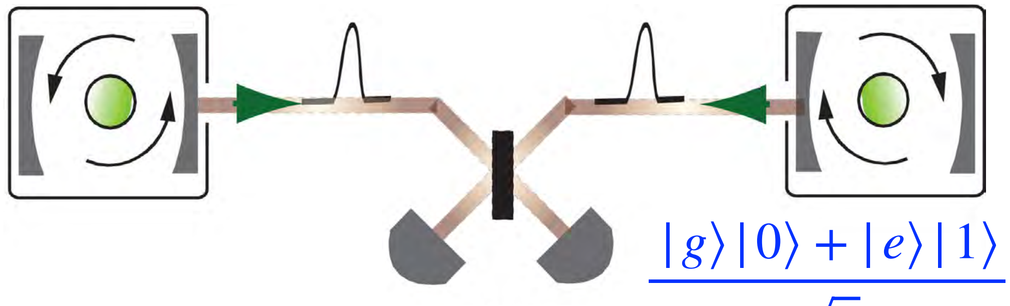
LHS photon transmission

RHS photon transmission

Postselection

Can we do better

- Both matter qubit emit a fock state qubit $|0\rangle, |1\rangle$
- So we have



$$\frac{|g\rangle|0\rangle + |e\rangle|1\rangle}{\sqrt{2}}$$

$$|g\rangle|g\rangle|0\rangle|0\rangle + |g\rangle|e\rangle|0\rangle|1\rangle + |e\rangle|g\rangle|1\rangle|0\rangle + |e\rangle|e\rangle|1\rangle|1\rangle$$

50/50 BS

$$\{|e\rangle|g\rangle - |g\rangle|e\rangle\}\boxed{|1\rangle|0\rangle} + \{|e\rangle|g\rangle + |g\rangle|e\rangle\}\boxed{|0\rangle|1\rangle}$$
$$+|g\rangle|g\rangle|0\rangle|0\rangle + |e\rangle|e\rangle\{|2\rangle|0\rangle - |0\rangle|2\rangle\}$$

Measure 1 photon

$$|g\rangle|e\rangle \pm |e\rangle|g\rangle$$

$$P = \sqrt{\eta}/2$$

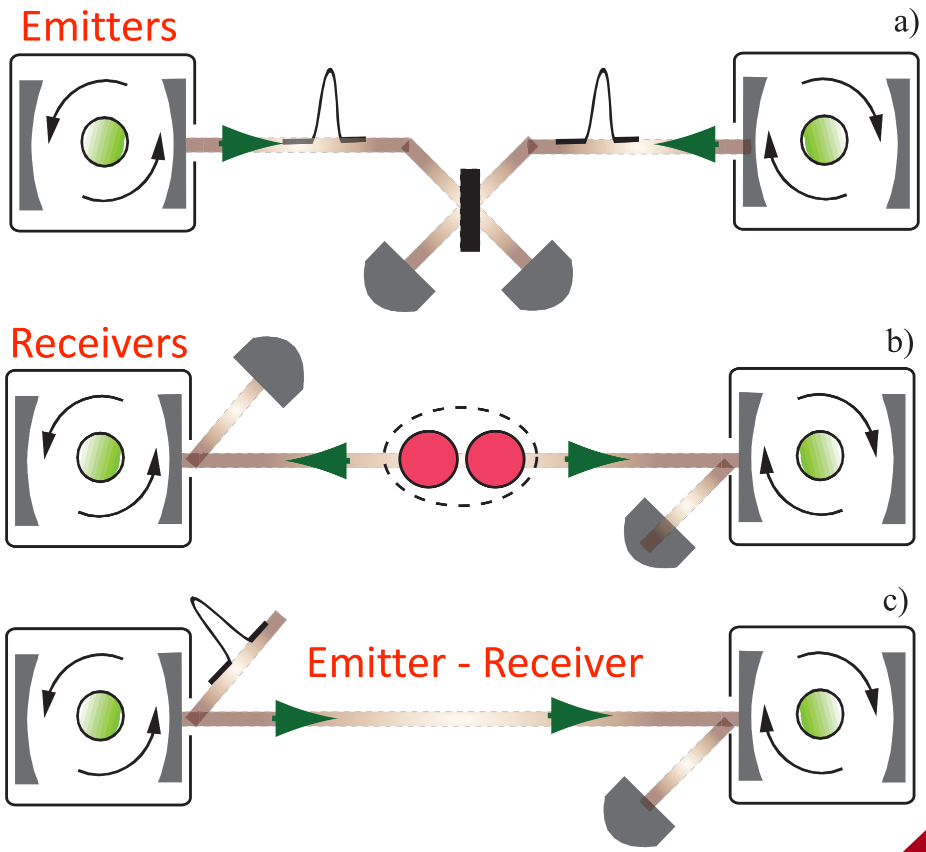
Entanglement distribution

Three conceptual schemes

- All are probabilistic - but heralded so we know when they work
- That heralding requires classical communication between the nodes

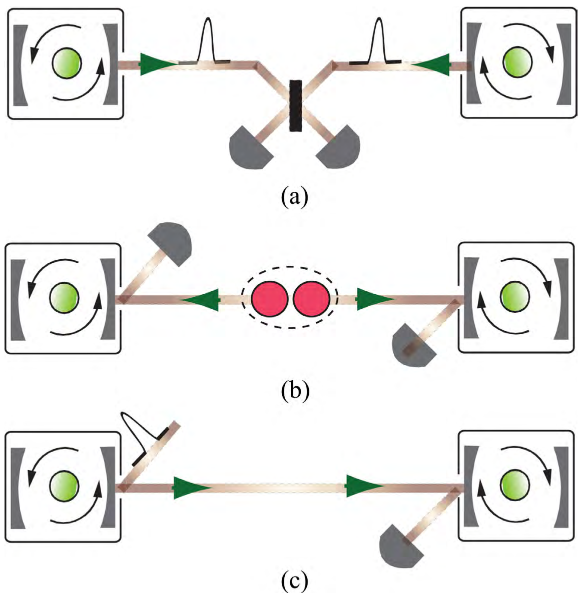
- Emitter - Node emits a photon entangled with memory qubit.

- Reciever - receives an incoming photon and entails it with the matter qubit



Entanglement Distribution

- Encodings:
  - Single-rail: using the presence or absence of a single photon in a single mode
  - Dual-rail: the presence of a photon in one or another mode as the qubit
    - Do not include the vacuum
  - Three basic types



$$\eta = e^{-L_{tot}/L}$$

Scaling?

S	D
$\sqrt{\eta}$	η
$\sqrt{\eta}$	η
$\eta?$	η

Which is better?

- Single-Rail Encoding:
  - Pros: Simpler
  - Cons: Loss-sensitive
- Dual-Rail Encoding:
  - Pros: More robust
  - Cons: shorter distances
  - Common in photonic experiments

Can be extended to qudits!!!

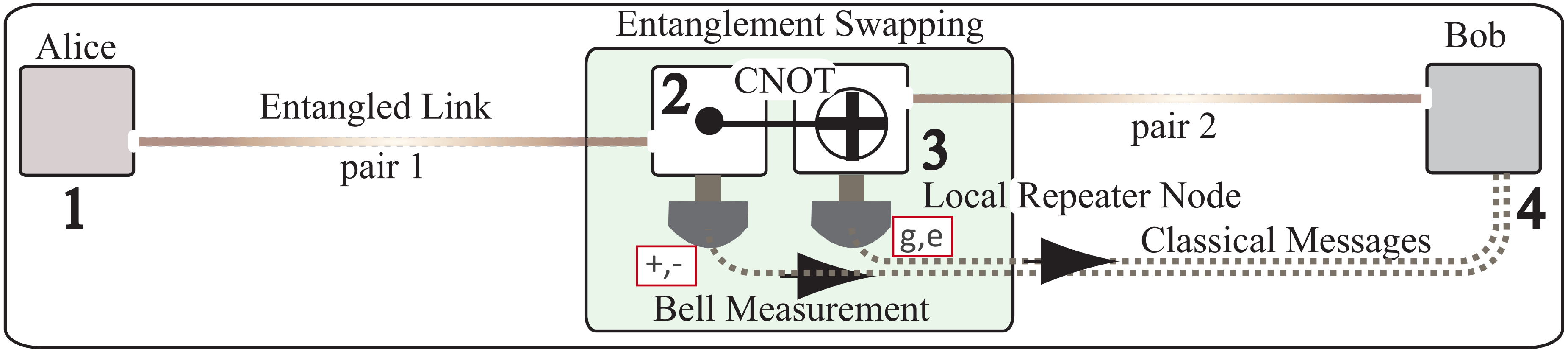
Entanglement Distribution

- Metrics and Performance
    - Fidelity
    - Entanglement Rate / Throughput
    - Success Probability / Yield
    - Entanglement Length
    - Entanglement Cost
    - Quantum Bit Error Rate (QBER)

- Why Metrics Matter:
    - They define the reliability and usefulness of quantum links.

Entanglement Swapping

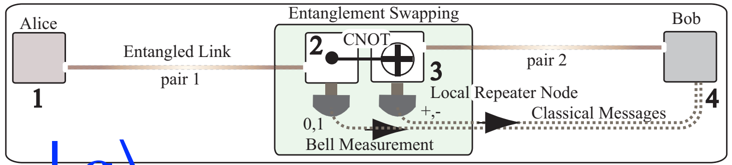
- The basic concept is to perform a Bell state measurement (CNOT plus measurement on the qubits within one repeater node



- Consider

$$\{\,|g\rangle_A|g\rangle_N + |e\rangle_A|e\rangle_N\} \otimes \{\,|g\rangle_N|g\rangle_B + |e\rangle_N|e\rangle_B\}$$

Entanglement Swapping



$$|g\rangle_A |g\rangle_N |g\rangle_N |g\rangle_B \xrightarrow{\text{CNOT}} |g\rangle_A |g\rangle_N |g\rangle_N |g\rangle_B$$
$$|g\rangle_A |g\rangle_N |e\rangle_N |e\rangle_B \rightarrow |g\rangle_A |g\rangle_N |e\rangle_N |e\rangle_B$$
$$|e\rangle_A |e\rangle_N |g\rangle_N |g\rangle_B \rightarrow |e\rangle_A |e\rangle_N |e\rangle_N |g\rangle_B$$
$$|e\rangle_A |e\rangle_N |e\rangle_N |e\rangle_B \rightarrow |e\rangle_A |e\rangle_N |g\rangle_N |e\rangle_B$$

- Measure qubit 3 in g, e basis

$|g\rangle_A |g\rangle_N |g\rangle_B + |e\rangle_A |e\rangle_N |e\rangle_B$ for g

$|g\rangle_A |g\rangle_N |e\rangle_B + |e\rangle_A |e\rangle_N |g\rangle_B$ for e

- Measure qubit 2 in +,- basis

$|g\rangle_A |g\rangle_B + |e\rangle_A |e\rangle_B$ for qubit 2 = +, qubit 3 = g

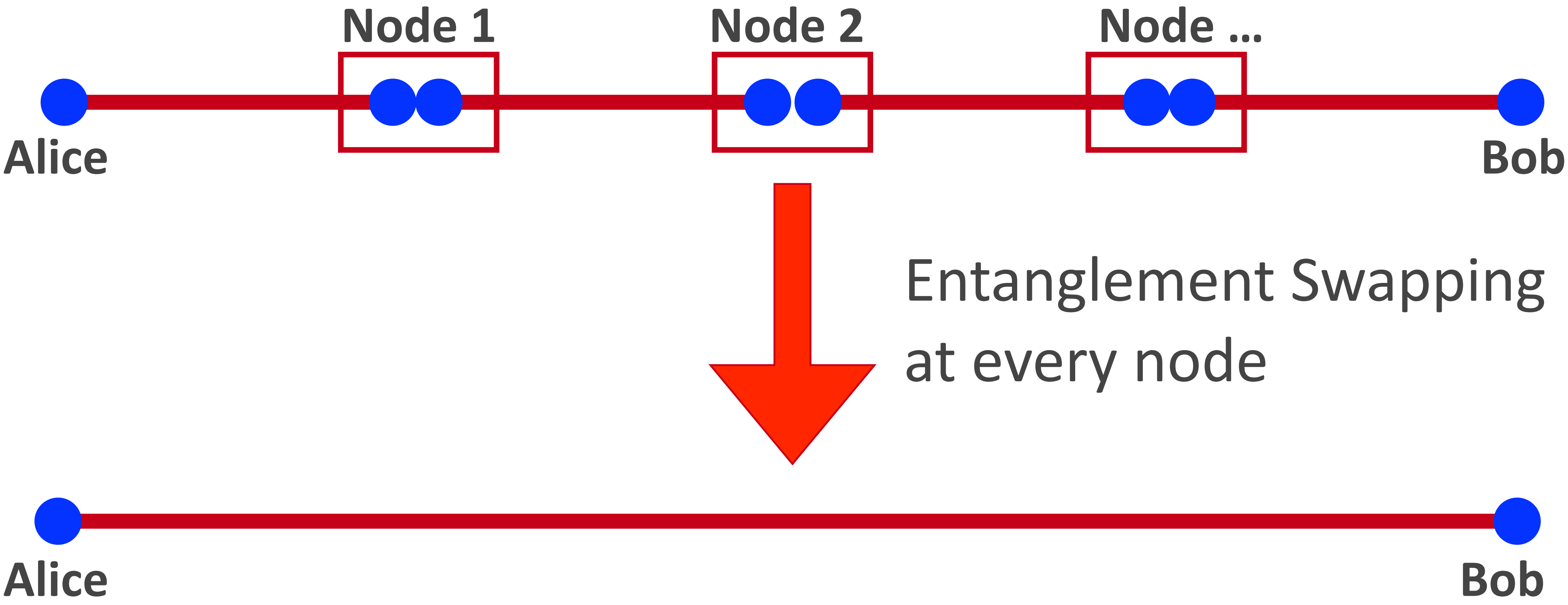
$|g\rangle_A |g\rangle_B - |e\rangle_A |e\rangle_B$ for qubit 2 = -, qubit 3 = g

$|g\rangle_A |e\rangle_B + |e\rangle_A |g\rangle_B$ for qubit 2 = +, qubit 3 = e

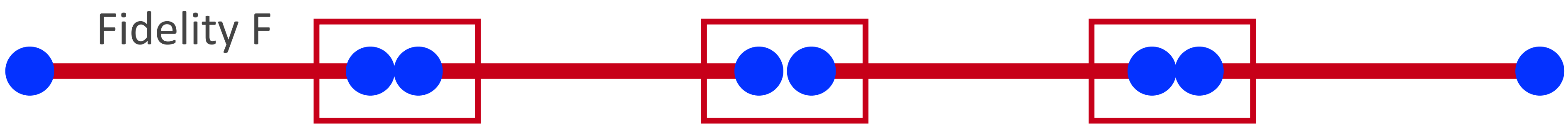
$|g\rangle_A |e\rangle_B - |e\rangle_A |g\rangle_B$ for qubit 2 = -, qubit 3 = e

So swapping gives us a Bell pairs between Alice and Bob

Swapping down the chain



Quantum Relay



$$p_s = e^{-L/nL_0}$$

- **This allows us to establish long range entanglement efficiently**

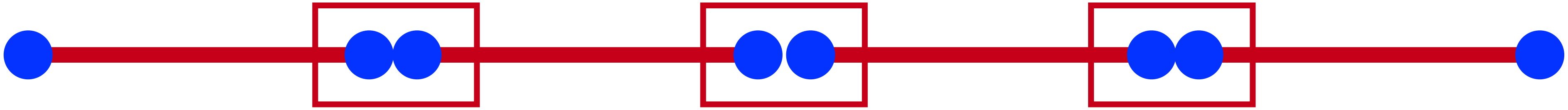
- So what is the problem?
  - The quality (fidelity) of the long range Bell pairs scales as

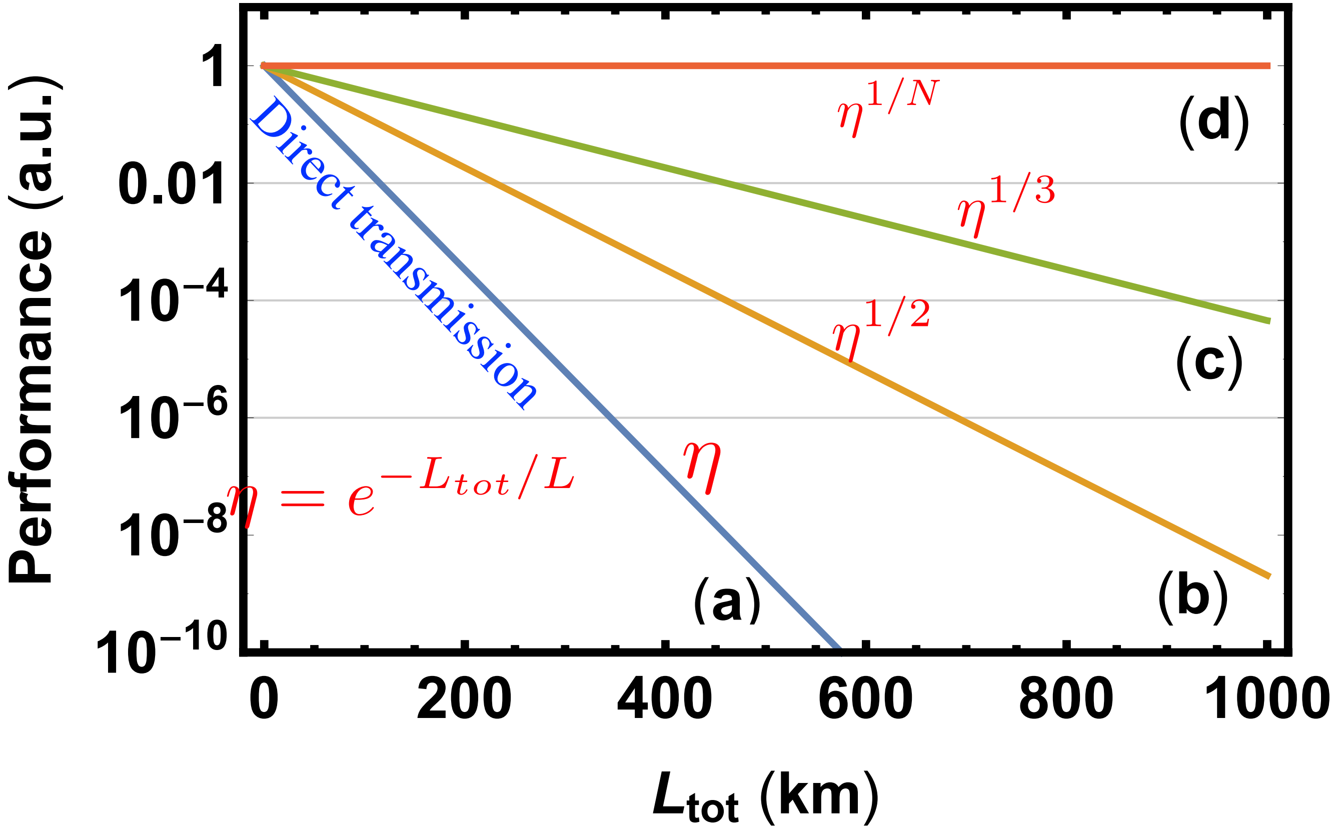
$$F_S \sim F^{n-1}$$

it decreases with the number of nodes due to noise

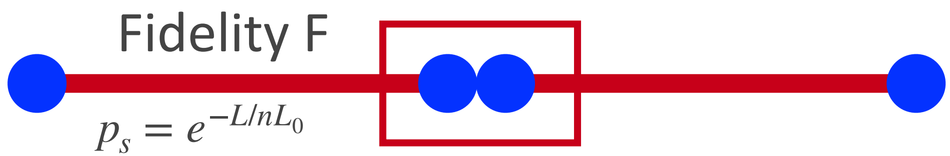
We need to fix this!!!

Quantum Relay Performance





Imperfections



- What happens if we don't have the perfect Bell state generated?
- Consider $\rho = F|\Phi_+\rangle\langle\Phi_+| + (1-F)|\Psi_+\rangle\langle\Psi_+|$ with $|\Phi_\pm\rangle = |00\rangle \pm |11\rangle$, $|\Psi_\pm\rangle = |01\rangle \pm |10\rangle$

- Our initial state for swapping can be written $\rho_{AN} \otimes \rho_{NB} =$

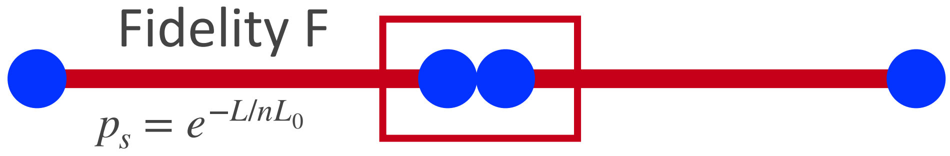
$$
\begin{aligned}
F^2 &: \quad |\Phi_+\rangle\langle\Phi_+| \otimes |\Phi_+\rangle\langle\Phi_+| \longrightarrow |\Phi_+\rangle\langle\Phi_+| \\
+F(1-F) &: \quad |\Phi_+\rangle\langle\Phi_+| \otimes |\Psi_+\rangle\langle\Psi_+| \longrightarrow |\Psi_+\rangle\langle\Psi_+| \\
+F(1-F) &: \quad |\Psi_+\rangle\langle\Psi_+| \otimes |\Phi_+\rangle\langle\Phi_+| \longrightarrow |\Psi_+\rangle\langle\Psi_+| \\
+(1-F)^2 &: \quad |\Psi_+\rangle\langle\Psi_+| \otimes |\Psi_+\rangle\langle\Psi_+| \longrightarrow |\Phi_+\rangle\langle\Phi_+|
\end{aligned}
$$

- Our resulting state between A and B has the form

$$\rho' = \boxed{[F^2 + (1-F)^2]}|\Phi_+\rangle\langle\Phi_+| + 2F(1-F)|\Psi_+\rangle\langle\Psi_+|$$

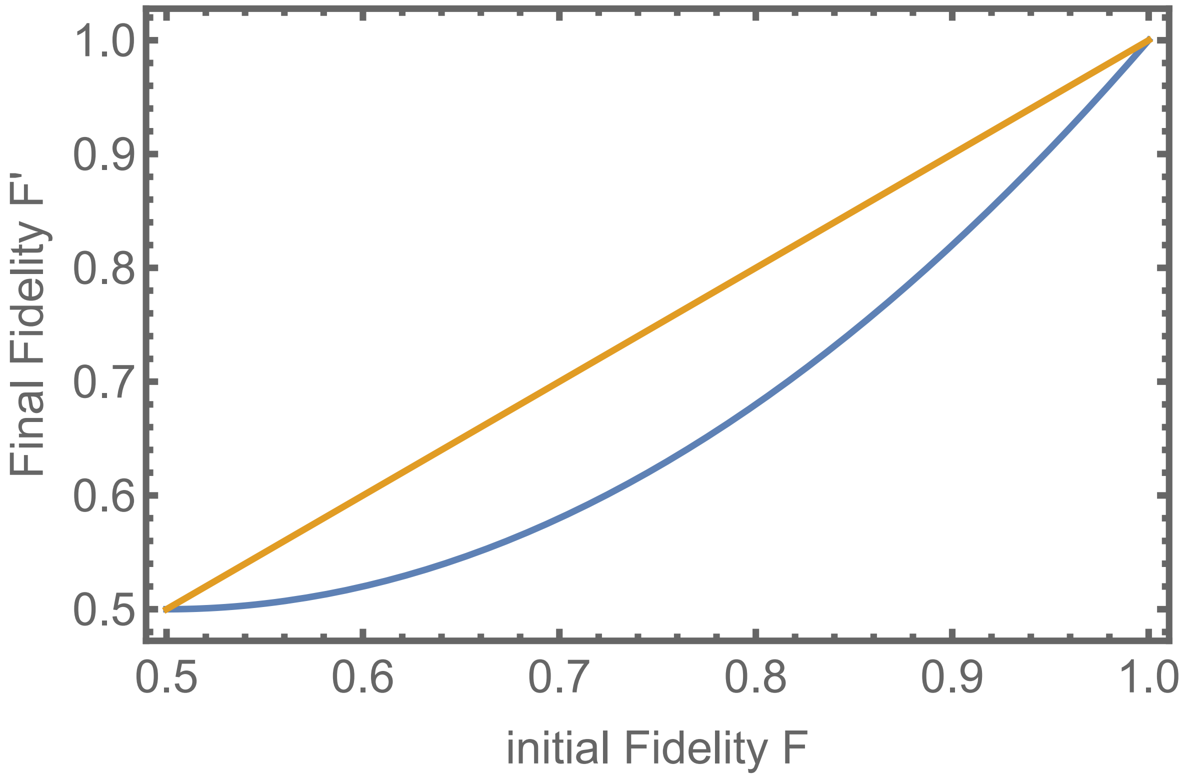
$\sim F^2$ for F close to one

Imperfections



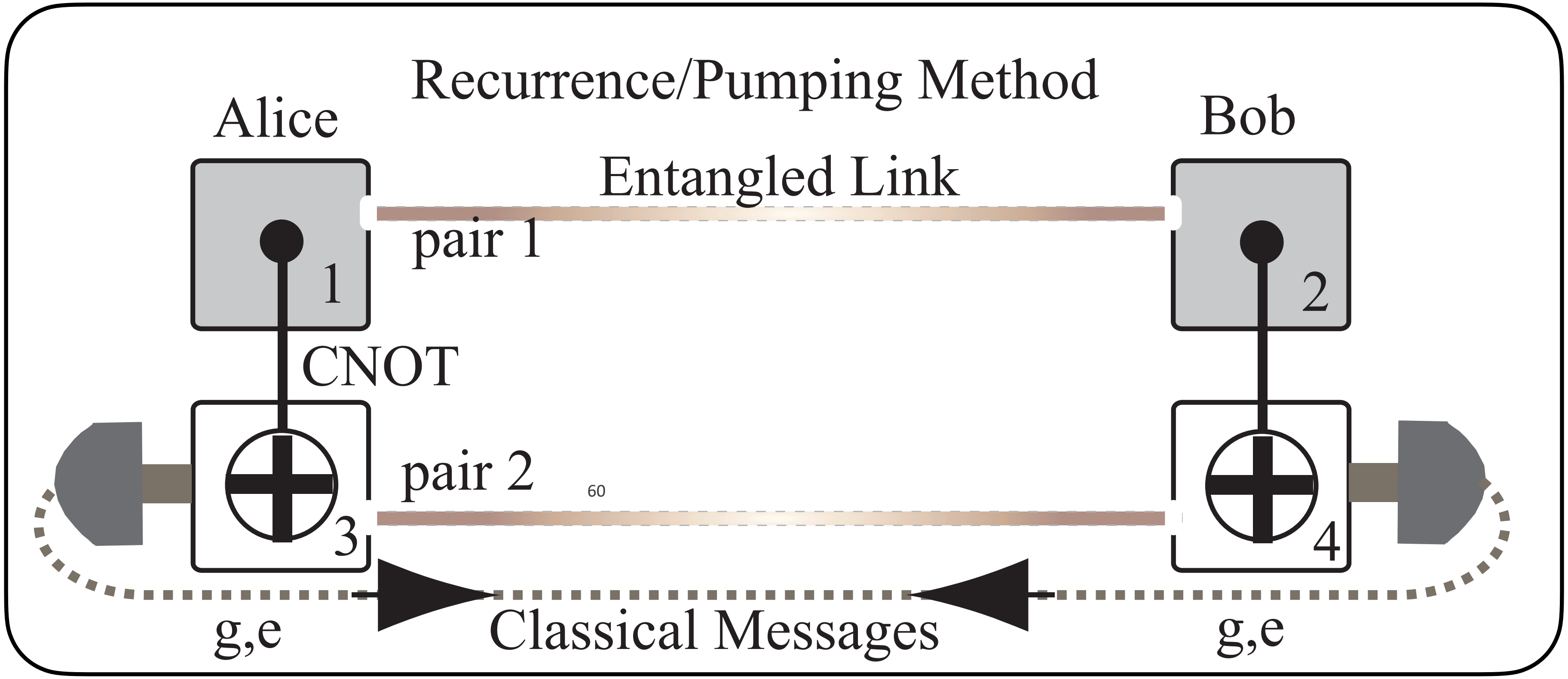
- Our resulting state between A and B has the form

$$\rho' = [F^2 + (1-F)^2]\,|\Phi_+\rangle\langle\Phi_+| + 2F(1-F)\,|\Psi_+\rangle\langle\Psi_+|$$

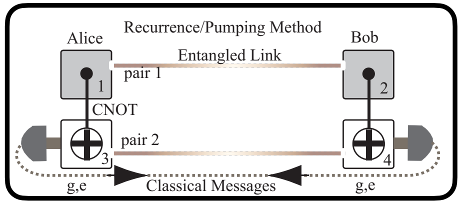


Entanglement Purification

- The basic concept is to take two imperfect Bell states and purify/distill them to one of higher quality.



An illustrative example



- Consider we have two copes of the state $\rho_{12} \otimes \rho_{1'2'}$ where $\rho = F|\Phi_+\rangle\langle\Phi_+| + (1-F)|\Psi_+\rangle\langle\Psi_+|$
- We apply a CNOT between (1 , 1') and (2,2')

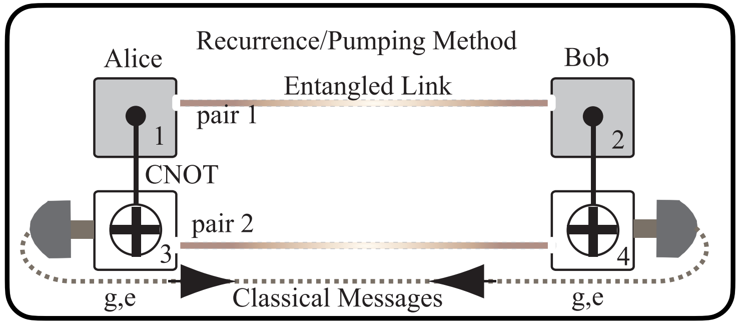
- Lets look at this piece by piece

$$|\Phi_+\rangle \otimes |\Phi_+\rangle = |gg+ee\rangle \otimes |gg+ee\rangle \longrightarrow |gg+ee\rangle \otimes |gg+ee\rangle = |\Phi_+\rangle \otimes |gg+ee\rangle$$

$$|\Phi_+\rangle \otimes |\Phi_+\rangle \longrightarrow |\Phi_+\rangle \otimes |gg+ee\rangle$$
$$|\Phi_+\rangle \otimes |\Psi_+\rangle \longrightarrow |\Phi_+\rangle \otimes |ge+eg\rangle$$
$$|\Psi_+\rangle \otimes |\Phi_+\rangle \longrightarrow |\Psi_+\rangle \otimes |ge+eg\rangle$$
$$|\Psi_+\rangle \otimes |\Psi_+\rangle \longrightarrow |\Psi_+\rangle \otimes |gg+ee\rangle$$

- So

$$\rho_{12} \otimes \rho_{1'2'} \longrightarrow \{F^2|\Phi_+\rangle\langle\Phi_+| + (1-F)^2|\Psi_+\rangle\langle\Psi_+|\} \otimes |\Phi_+\rangle\langle\Phi_+|$$
$$+F(1-F)\{|\Phi_+\rangle\langle\Phi_+| + |\Psi_+\rangle\langle\Psi_+|\} \otimes |\Psi_+\rangle\langle\Psi_+|$$



An illustrative example

- After the two CNOT we have the state

$$\rho_{12} \otimes \rho_{1'2'} \longrightarrow \{F^2 |\Phi_+\rangle\langle\Phi_+| + (1-F)^2 |\Psi_+\rangle\langle\Psi_+|\} \otimes |\Phi_+\rangle\langle\Phi_+| \\ +F(1-F)\{|\Phi_+\rangle\langle\Phi_+| + |\Psi_+\rangle\langle\Psi_+|\} \otimes |\Psi_+\rangle\langle\Psi_+|$$

- Now measurement qubit 1' and 2' in the g, e basis remembering

$$|\Phi_\pm\rangle = |00\rangle \pm |11\rangle \qquad |\Psi_\pm\rangle = |01\rangle \pm |10\rangle$$

- For gg or ee we have state $\rho'_{12} \longrightarrow F^2 |\Phi_+\rangle\langle\Phi_+| + (1-F)^2 |\Psi_+\rangle\langle\Psi_+|$
- For ge or eg we have state $\rho'_{12} \longrightarrow F(1-F)\{|\Phi_+\rangle\langle\Phi_+| + |\Psi_+\rangle\langle\Psi_+|\}$

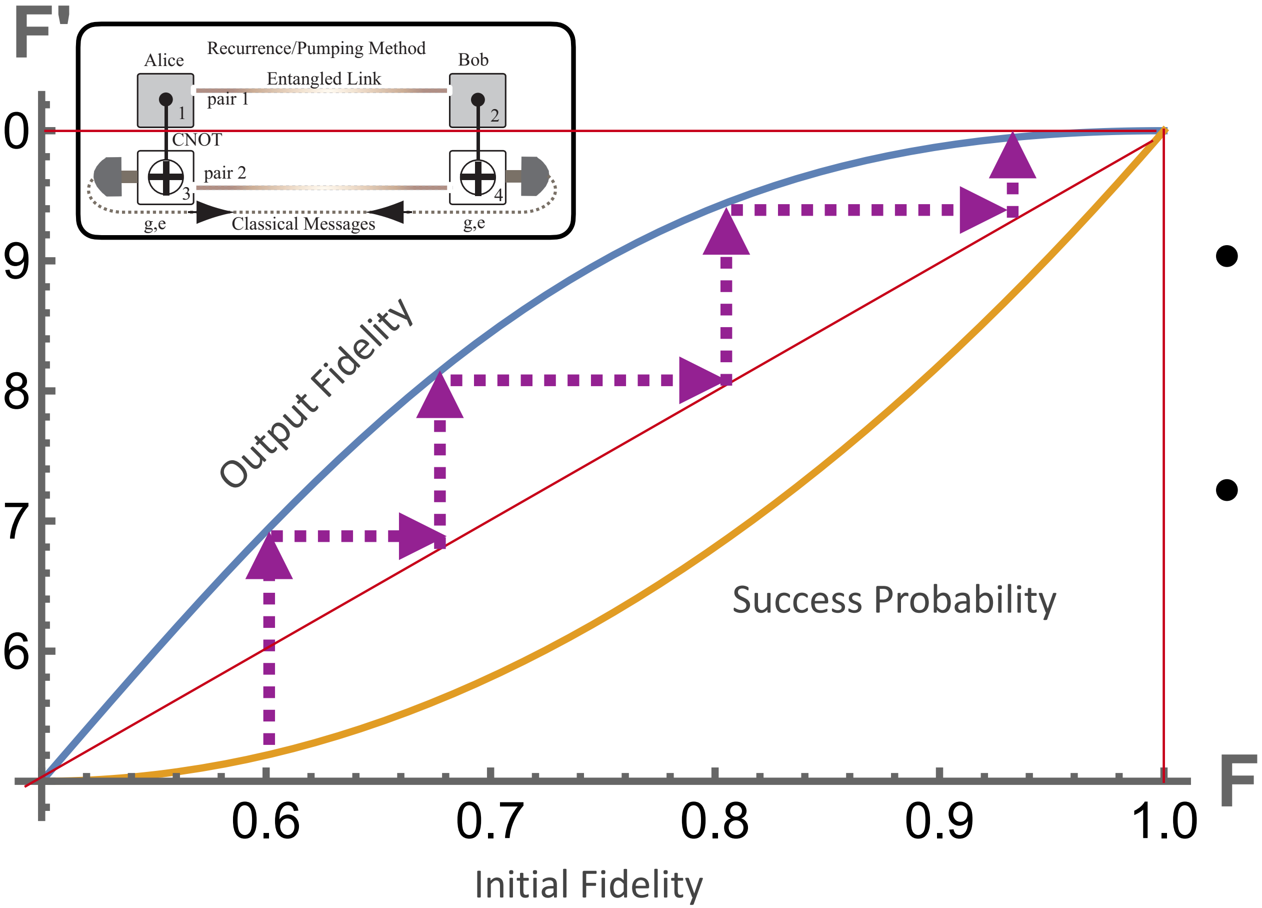
- Classical communication of measurements and keep only gg or ee

$$\rho'_{12} \longrightarrow F^2 |\Phi_+\rangle\langle\Phi_+| + (1-F)^2 |\Psi_+\rangle\langle\Psi_+|$$

An illustrative example

- The probability of measuring gg or ee is $F^2 + (1-F)^2$
  - Need to normalize the state $\rho'_{12} = F'|\Phi_+\rangle\langle\Phi_+| + (1-F')|\Psi_+\rangle\langle\Psi_+|$

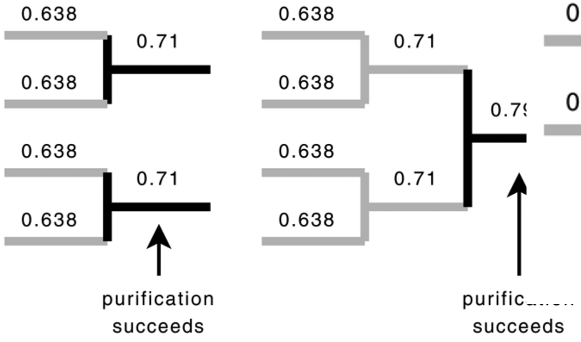
where $F' = \dfrac{F^2}{F^2 + (1-F)^2}$



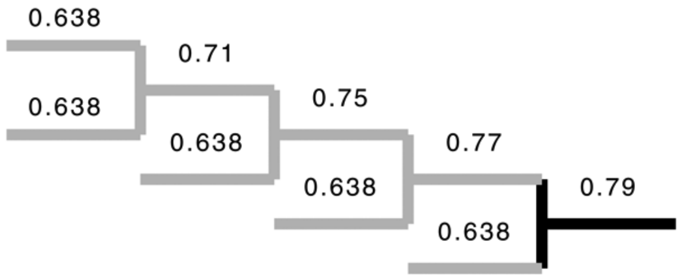
- So with many rounds of purification, we can get a high fidelity Bell pair.
- It consumes time because of the classical communication

Purification Scheduling

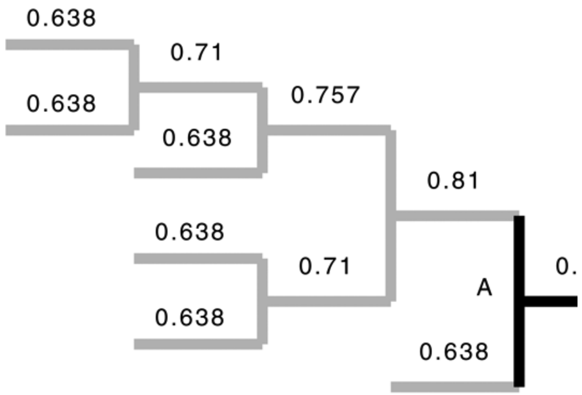
- What happens if you available Bell pairs are not all the same
- There is generally a trade of between link generation rate and fidelity



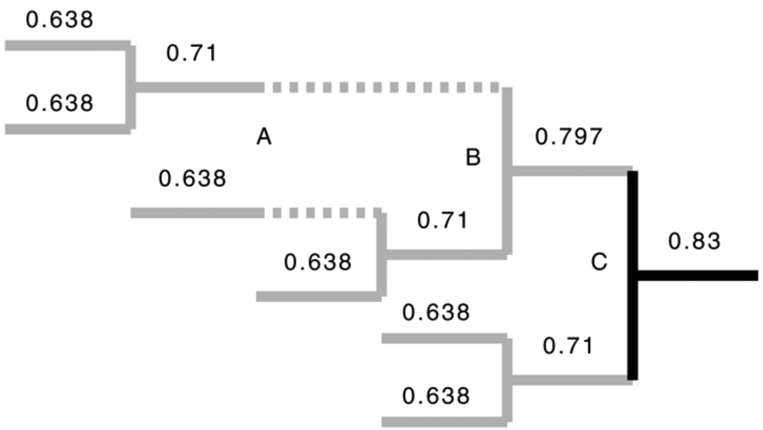
Symmetric



Pumping

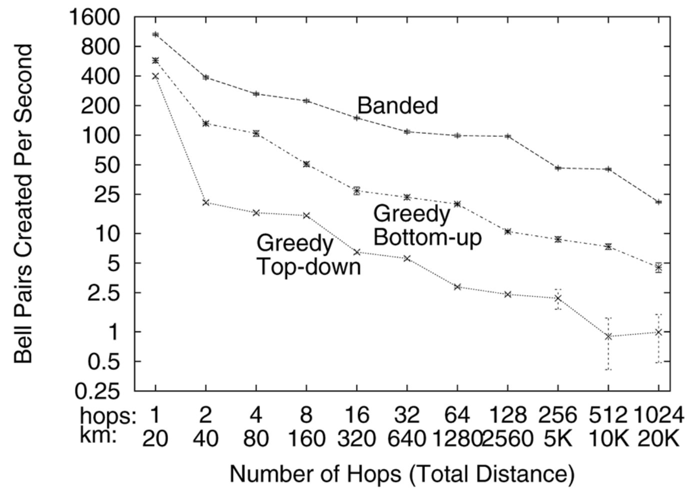


Greedy

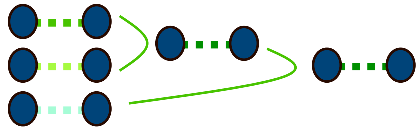


Banded

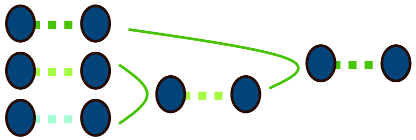
IEEE/ACM Transactions on Networking 17, 1002-1013 (2009).



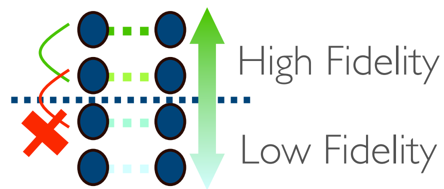
Greedy Top Down:



Greedy Bottom Up:

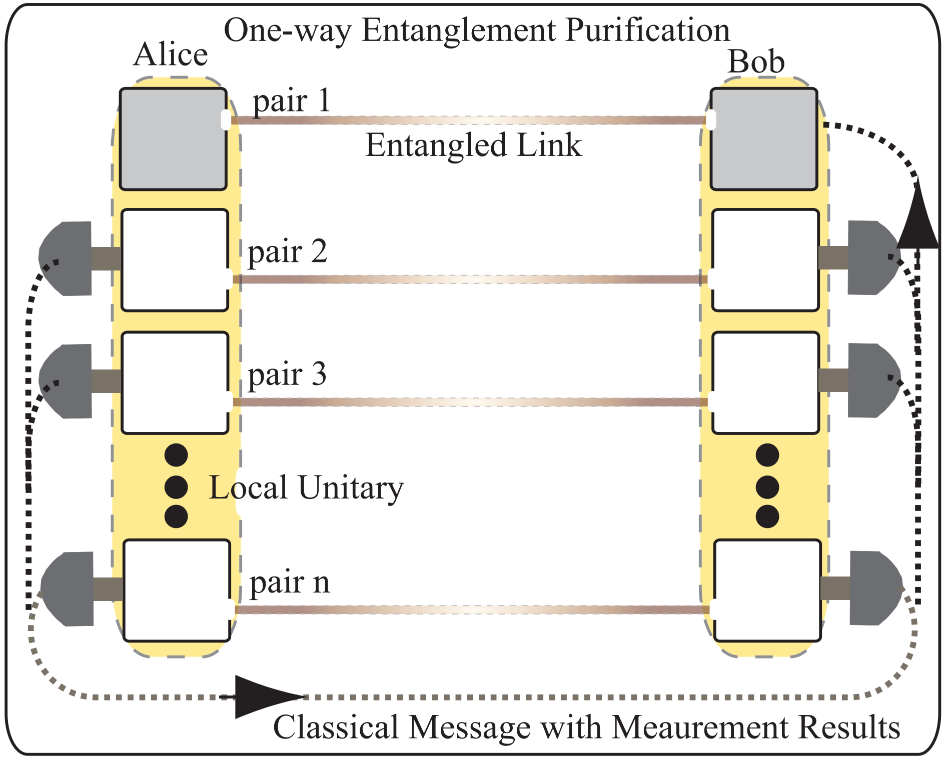


Banded:

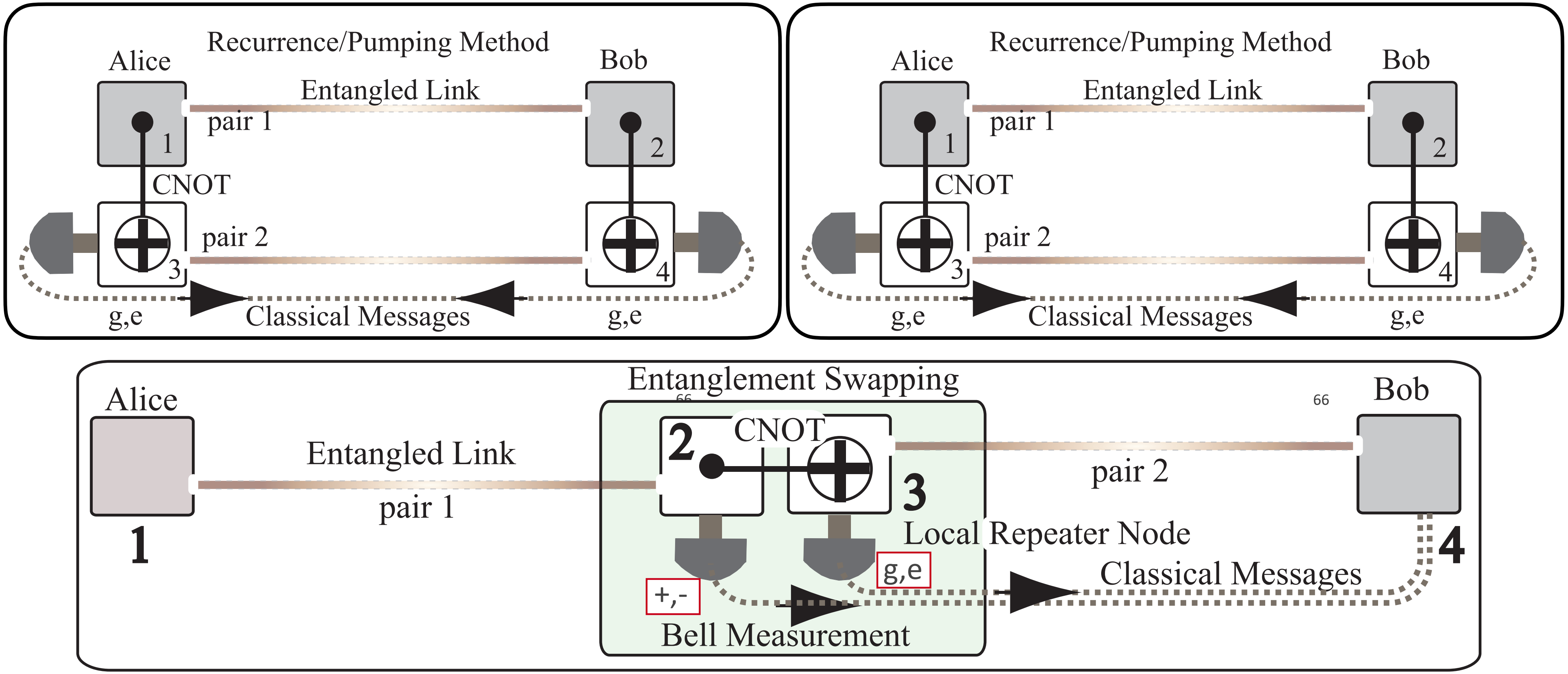


One Step Quantum Purification

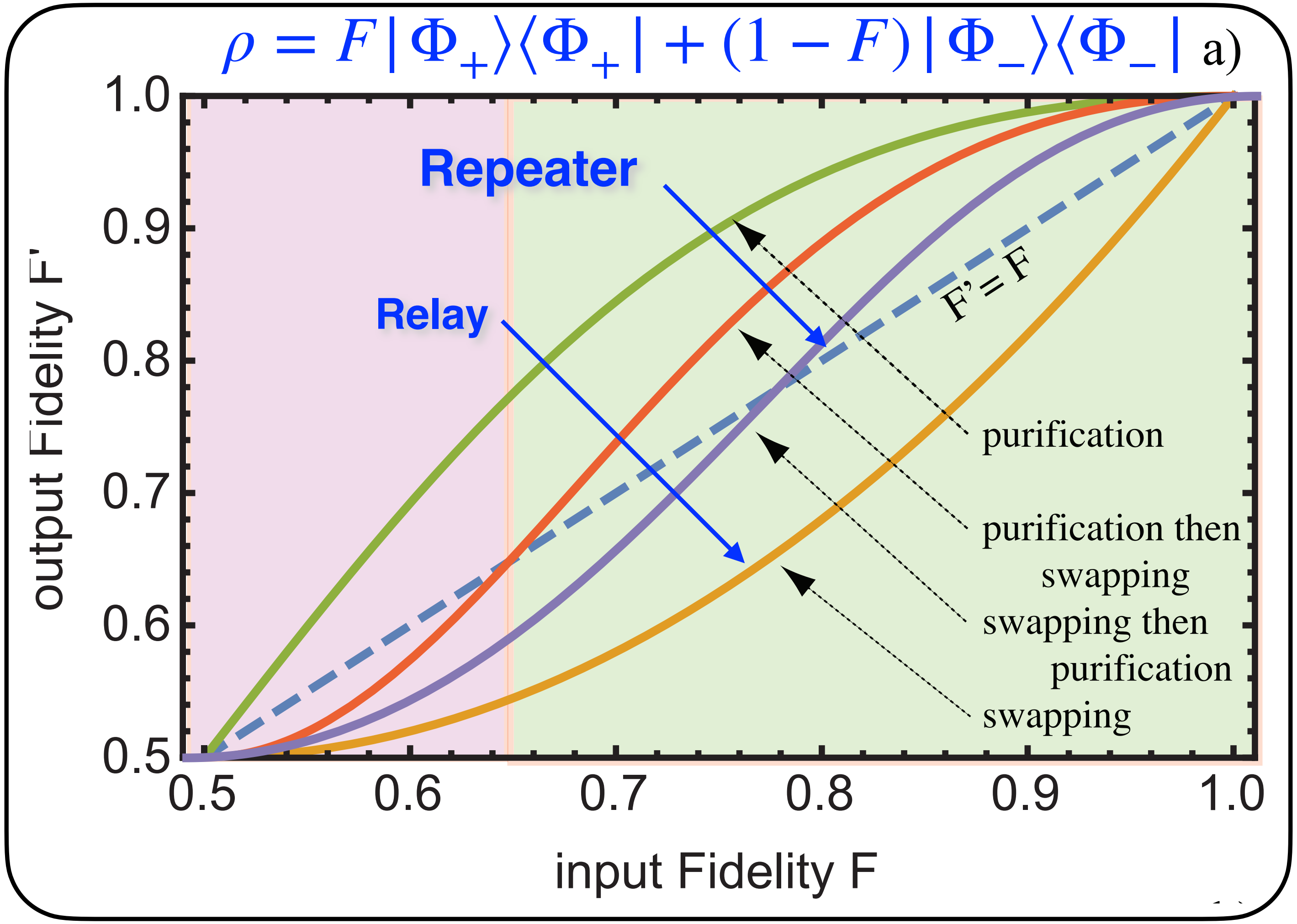
- The problem with most purification protocols is that there are potentially many rounds of purification required.
- We can think about using quantum error detection codes (still probabilistic but reduces the number of purification rounds)



Entanglement Purification and Swapping



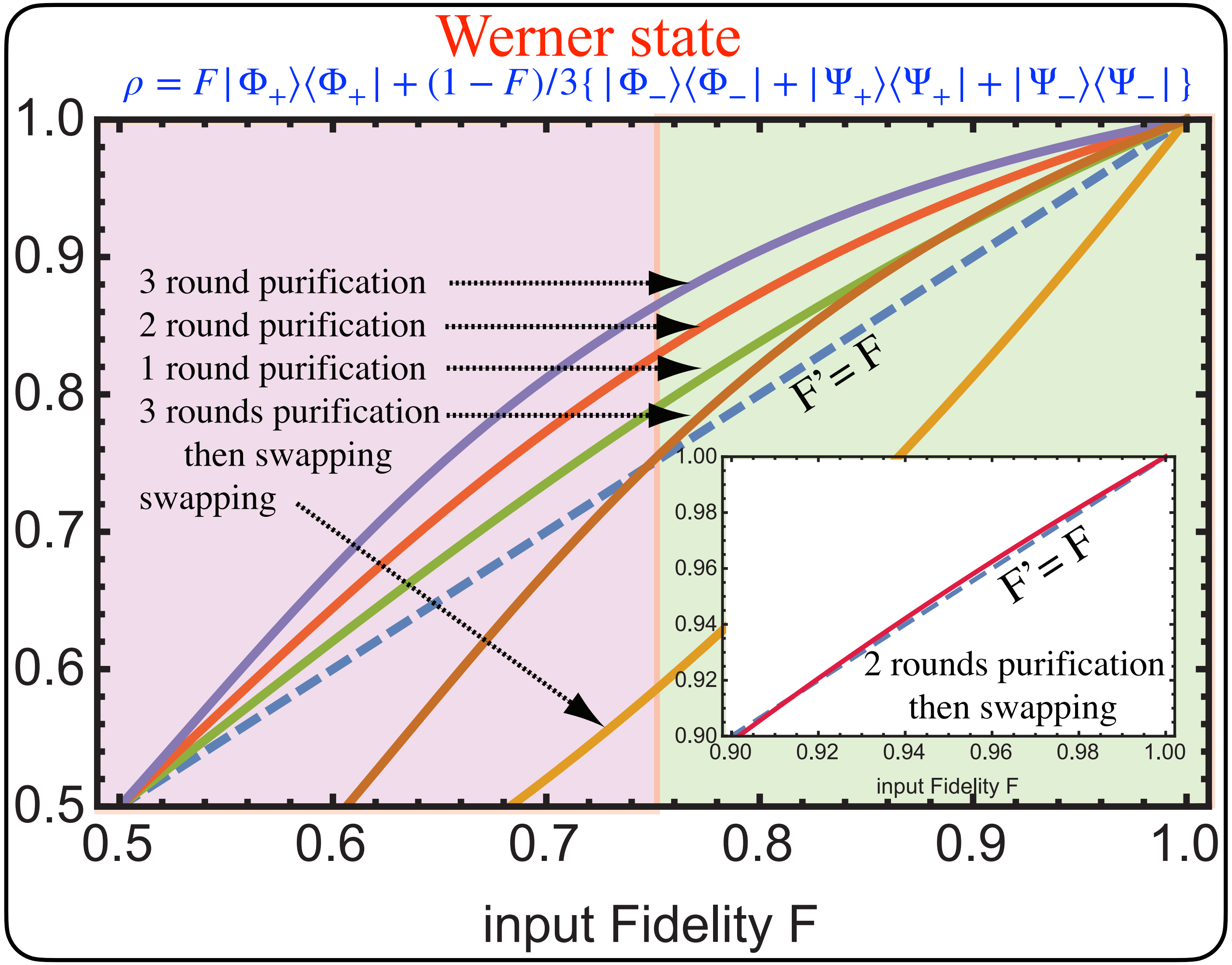
Entanglement Purification and Swapping together



- The maximum fidelity after purification is $1 - 2\epsilon_{\text{gate}}$

Entanglement Purification and Swapping together

- The form of the imperfect Bell state has a profound effect on the number of rounds of purification required



Essential Quantum Components

So what do we need to make a quantum repeater

- Single or entangled photon source
- Single photon detector
- Quantum Memory
- Quantum Buffer
- Frequency convertor
- Linear optical elements
- …

Single Photon Sources and Detectors

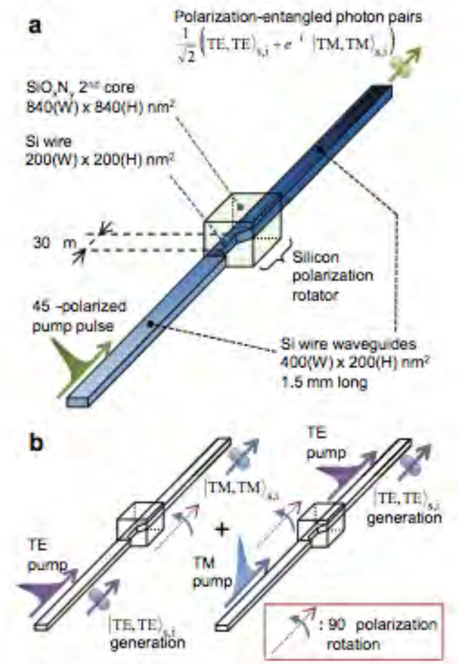
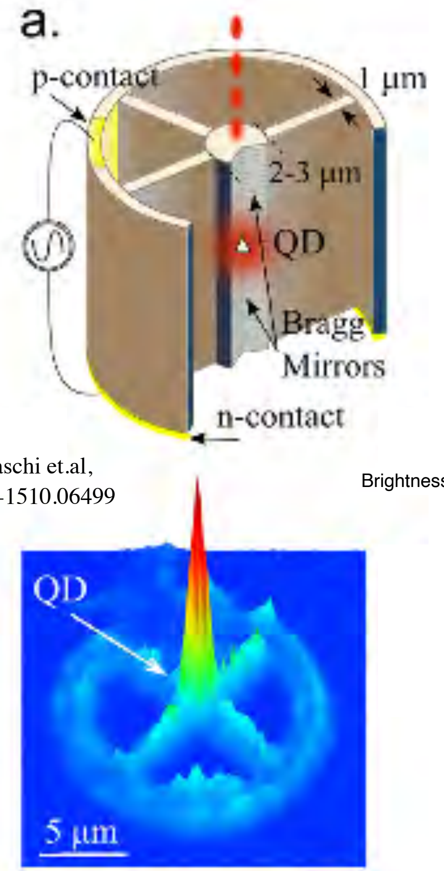


Figure 1 | A monolithically integrated polarization-entanglement source. (a) The source, fabricated on a silicon-on-insulator substrate, consists of a silicon-wire-based 90° polarization rotator sandwiched by two nonlinear silicon wire waveguides. The device generates the polarization entanglement as a superposition of the two events shown in (b). The figure is not to scale for clarity.



N. Somaschi et.al, quantph-1510.06499

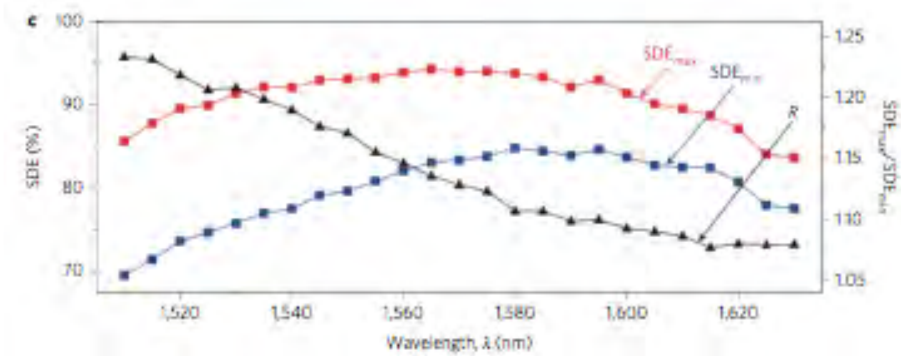
Brightness ~ 20%

Fig.1: Electrically controlled single photon sources: a: Schematic of the devices under study: a micropillar coupled to a QD is connected to a surrounding circular frame by four one-dimensional wires. The top p-contact is defined on a large mesa adjacent to the frame. The n-contact is deposited on the back of the sample. **b:** Optical microscope

LETTERS nature photonics
PUBLISHED ONLINE: 24 FEBRUARY 2013 | DOI: 10.1038/NPHOTON.2013.13

Detecting single infrared photons with 93% system efficiency

F. Marsili[1]*, V. B. Verma[1], J. A. Stern[2], S. Harrington[1], A. E. Lita[1], T. Gerrits[1], I. Vayshenker[1], B. Baek[1], M. D. Shaw[2], R. P. Mirin[1] and S. W. Nam[1]*



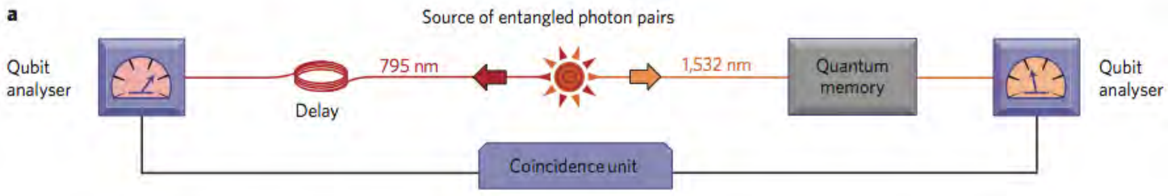
Quantum Memories

Developing well, but efficiencies still limited !!

nature photonics LETTERS
PUBLISHED ONLINE: 12 JANUARY 2015 | DOI: 10.1038/NPHOTON.2014.311

Quantum storage of entangled telecom-wavelength photons in an erbium-doped optical fibre

Erhan Saglamyurek[1], Jeongwan Jin[1], Varun B. Verma[2], Matthew D.
Sae Woo Nam[2], Daniel Oblak[1] and Wolfgang Tittel[1]*



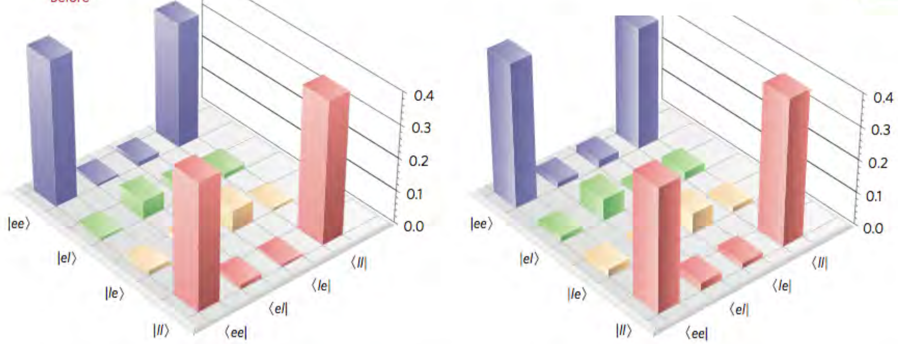
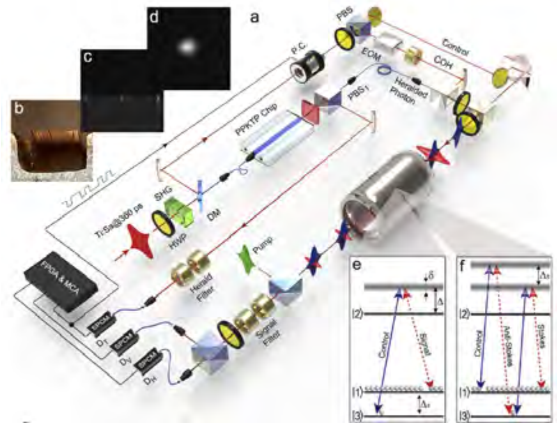


Figure 3 | Reconstructed density matrices. Measured density matrices of the photon pairs before (in) and after (out) storage of the telecom photon. Only the real components are shown (the absolute values of all imaginary components are below 0.025).

Table 1 | Characterization of the two-photon state.

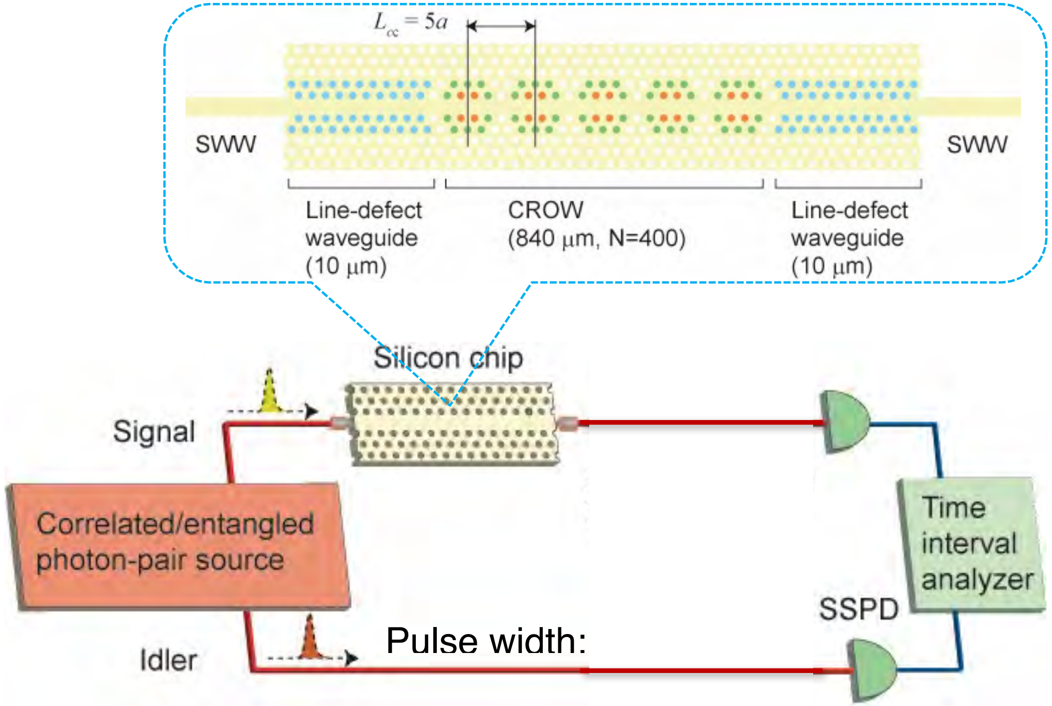
Quantity	Before storage	After storage
Fidelity with $\|\phi^+\rangle$ (%)	82.5 ± 0.4	80.8 ± 4.8
Purity (%)	69.4 ± 0.7	67.3 ± 4.7
Input/output fidelity (%)	97.1 ± 4.9	
Entanglement of formation (%)	53.1 ± 1.1	49.9 ± 10.5
Expected S_{th}	2.39 ± 0.01	2.35 ± 0.10
Measured S	2.38 ± 0.05	2.33 ± 0.22

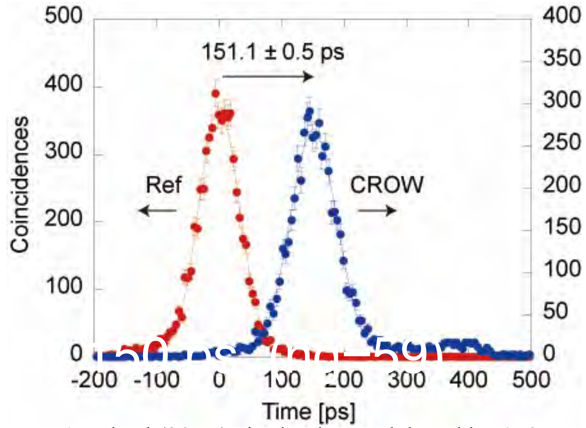
Interfacing GHz-bandwidth heralded single photons with a warm vapour Raman memory



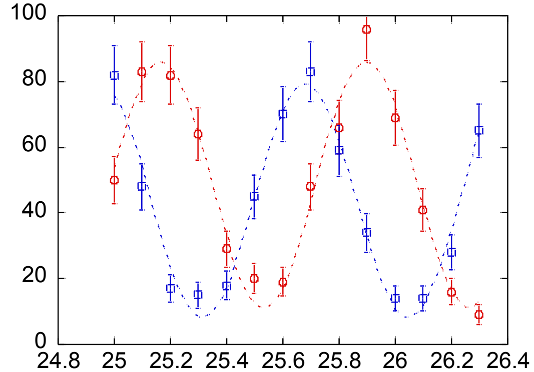
New J. Phys. 17 (2015) 043006

Quantum Buffers





A pulsed (20 ps) single photon delayed by 150 ps (ng=59) without major pulse distortion



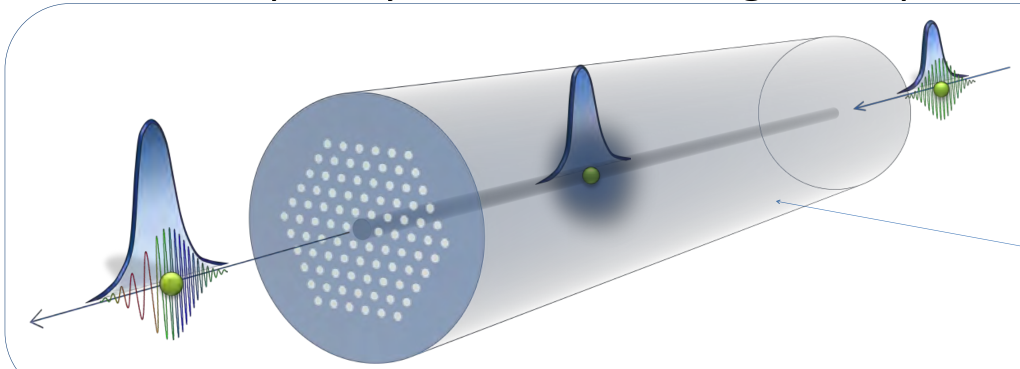
Preservation of quantum coherence

H. Takesue, et al Nature Communications 4, 2725 (2013).

A photon from a time-bin entangled state was stored and retrieved by CROW.

Frequency Converters

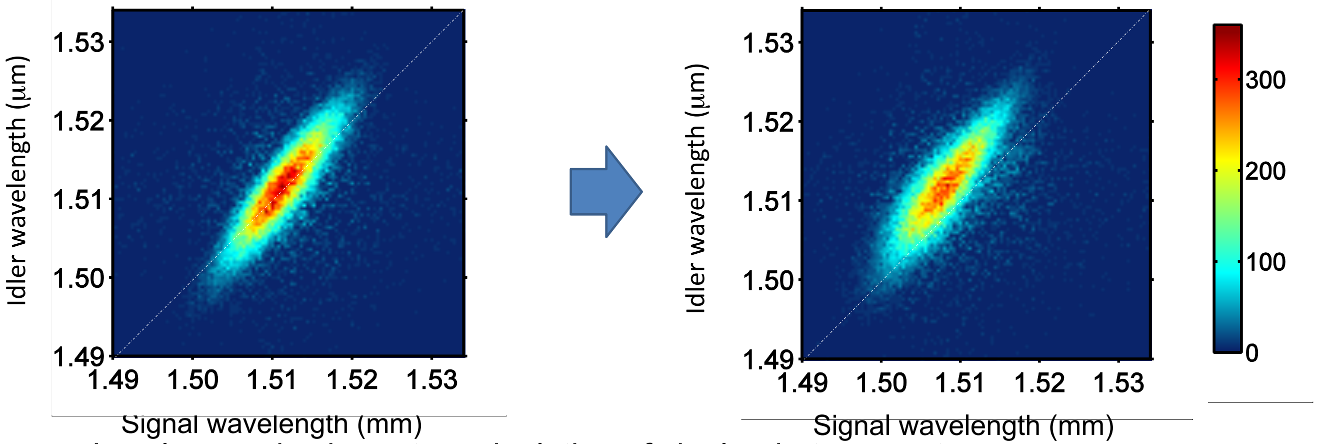
Photon frequency conversion using cross-phase modulation



Photonic crystal fiber

Joint spectral distribution of correlated photons

After the signal photons blue shifted

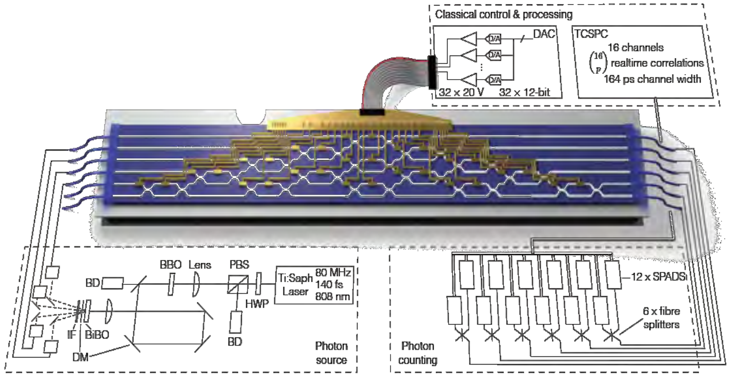


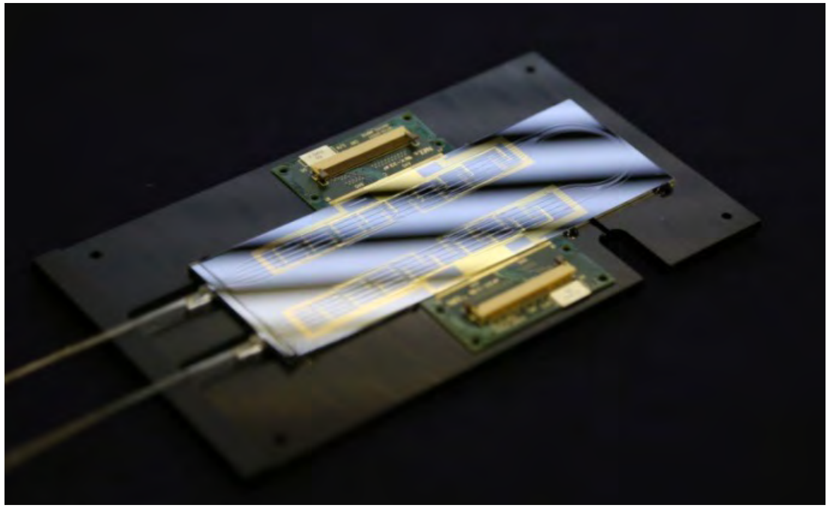
- Lossless and coherent manipulation of single-photon spectra

N. Matsuda, Sci. Adv. 2, e1501223

Simple quantum circuits

Universally-reconfigurable
linear-optics circuit

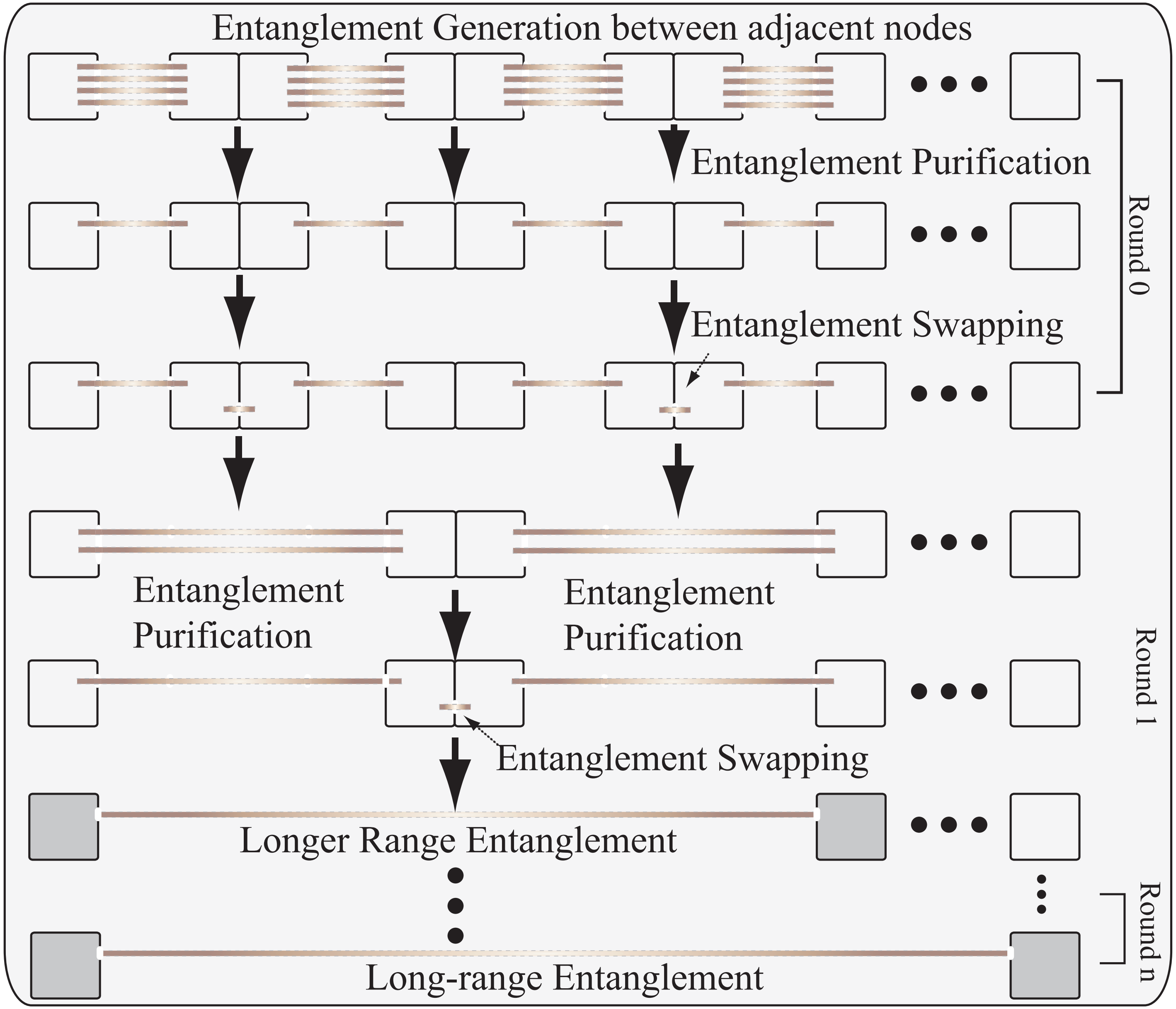




- Based on commercial silica-waveguide technology
- Comprised of 15 Mach-Zehnder interferometers and 30 thermo-optic phase shifters
- Arbitrary unitary transformation of path-encoded quantum states can be realized in a second
- Operation at 800 nm

J. Carolan *et al.*, Science **349**, 711 (2015).

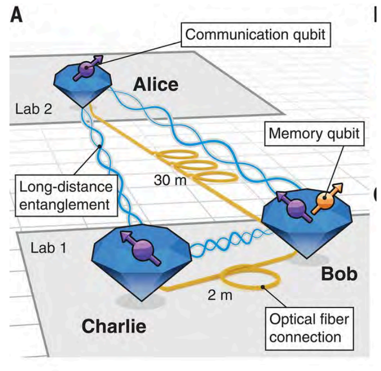
Conventional Quantum Repeaters



Matter and Photons

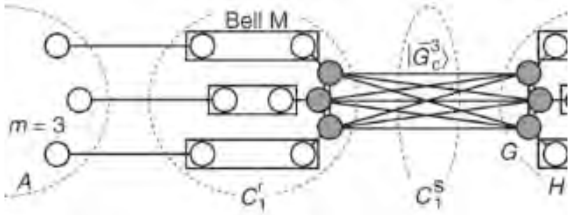
Matter-Based Quantum Networks

- Trapped ions, NV centers, Ensembles memories, …
  - Strengths: storage, local computation
  - Weaknesses: interface complexity with photons



Photonic Quantum Networks

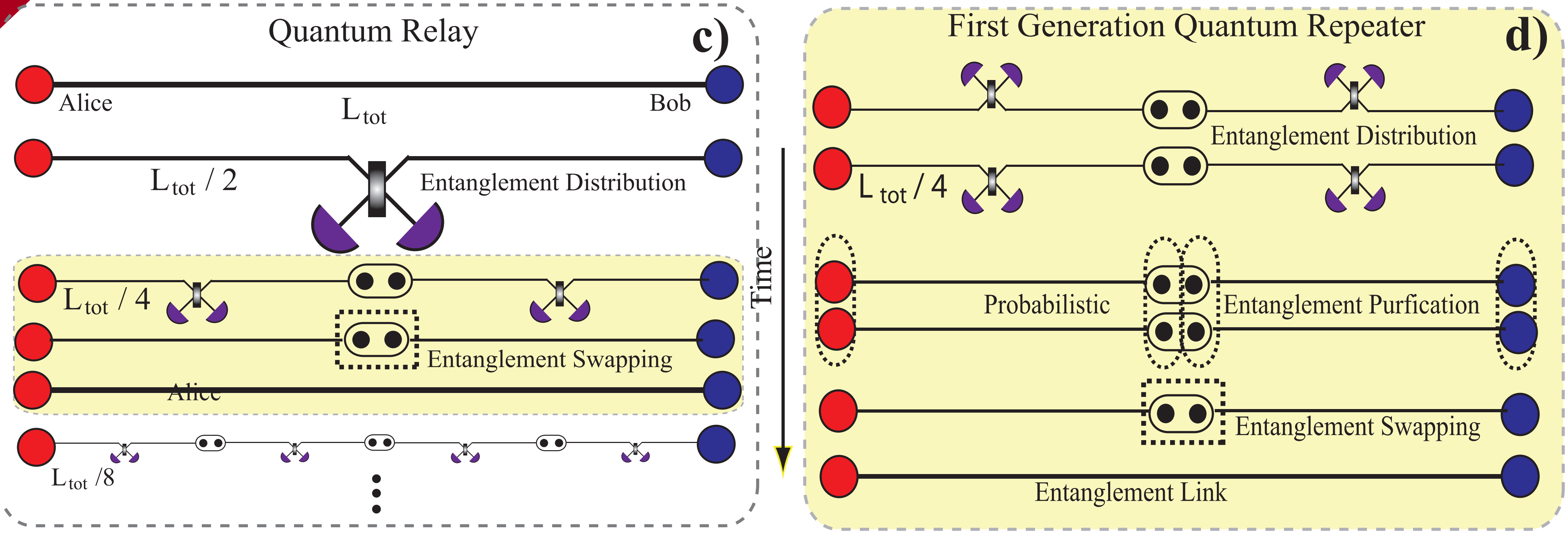
- Flying qubits: photons in fiber/free space
  - Strengths: long-distance, low decoherence
  - Challenges: photon loss, synchronization



Hybrid Quantum Networks

- Combine the two to leverage both strengths!!!

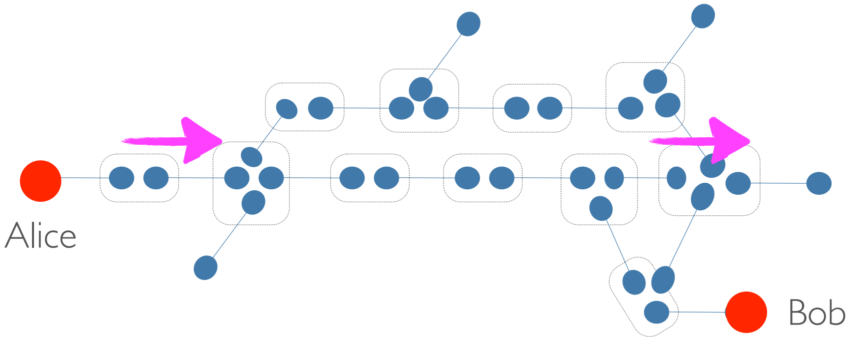
Quantum Relays vs Repeaters



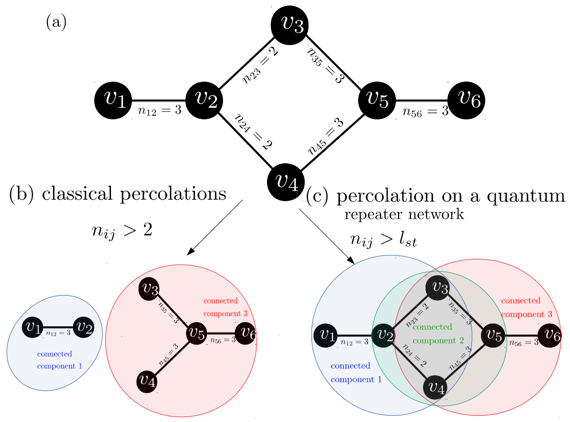
- 1st generation quantum repeaters and relay's look quite similar.
  - Both can use the same entanglement distribution and swapping operations.
  - Quantum repeater also include a purification mechanism

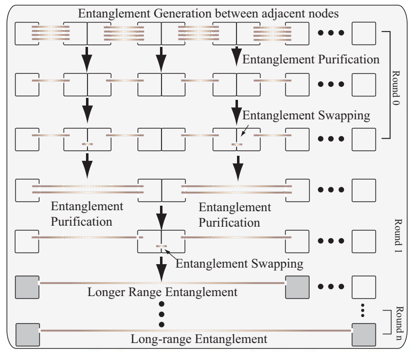
A warning: An unstable quantum network

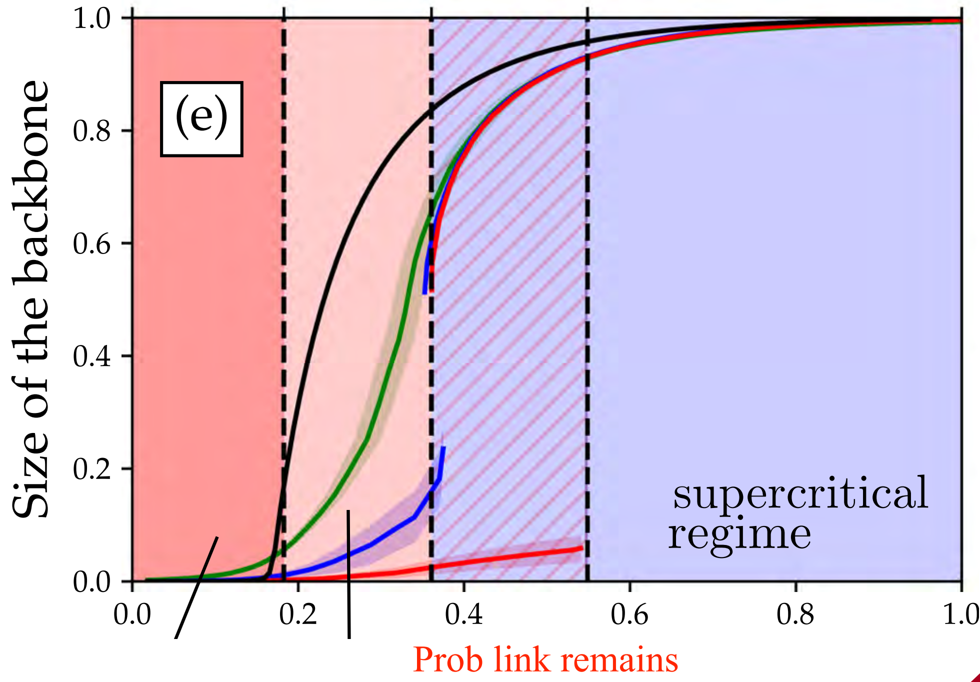
- What happens as our networks get larger?



- Probabilistic operations become a real problem!!!



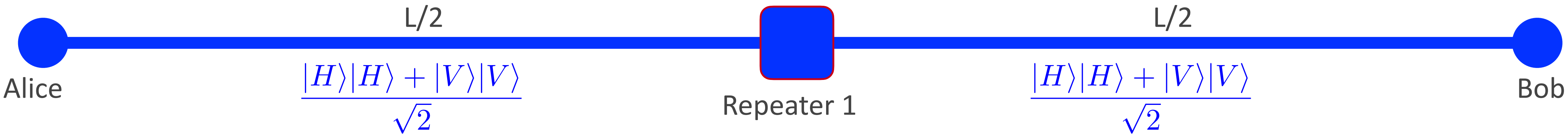




B.C. Coutinho et. al, Communications Physics 5, 105 (2022).

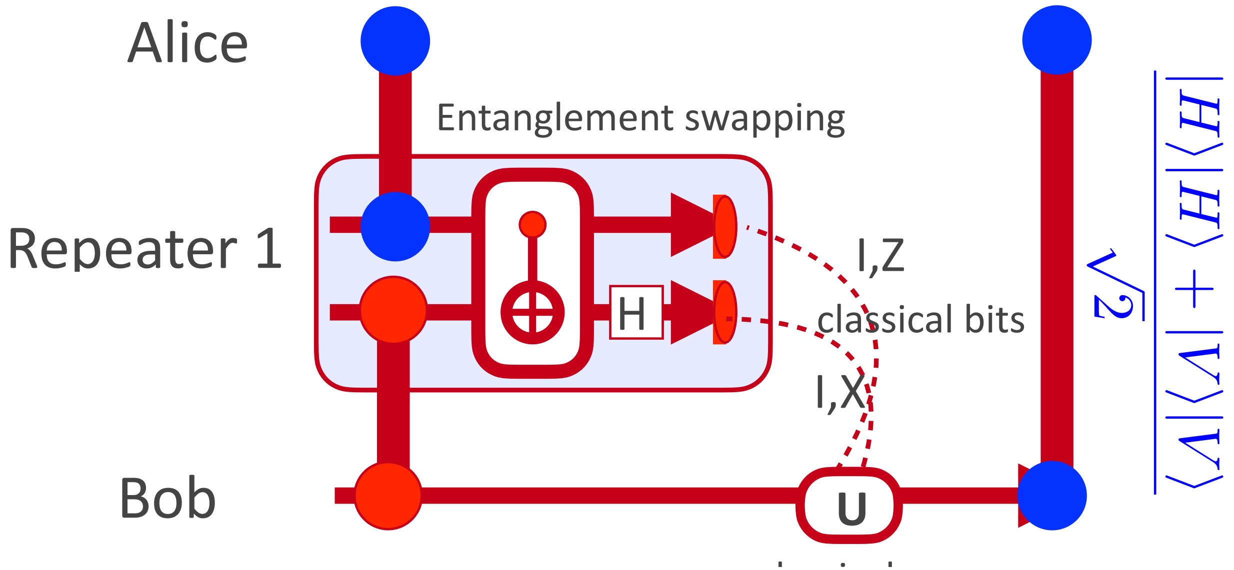
Summary: Quantum Repeater act differently

- First we can not use amplifers to correct for single loss
- So instead we take our channel and divide it into smaller links



We create the Bell state between all the adjacent nodes

(distance should be short enough that we can do this with low error when it works)



A Bell state at twice the distance

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